Mission Applications for Highly Non-Keplerian Orbits (3-body problem)

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University of Strathclyde Glasgow

Overview

- Solar sails are capable of a whole class of orbits beyond those of the traditional **conic section** (or Keplerian) orbits.
- These non-Keplerian orbits are one of the advantages of using solar sail technology.
- A particularly relevant, and interesting, setting to study non-Keplerian orbits is the **Earth-Sun 3-body problem**, which will be the focus of this talk.

Lagrange points

First some background on classical (conventional spacecraft) dynamics in the circular restricted 3-body problem:

- The CR3BP means:
  - Third body's mass is negligible compared to the primaries' (*restricted*).
  - Primaries orbit *circularly* in the (ecliptic) plane about their centre of mass.
- We non-dimensionalise units to set the following quantities to unity:
  - Constant of gravitation, G.
  - Sum of primaries' masses: $\mu + (1 - \mu) = 1$.
  - Distance between the primaries.
  - Angular velocity of primary masses.

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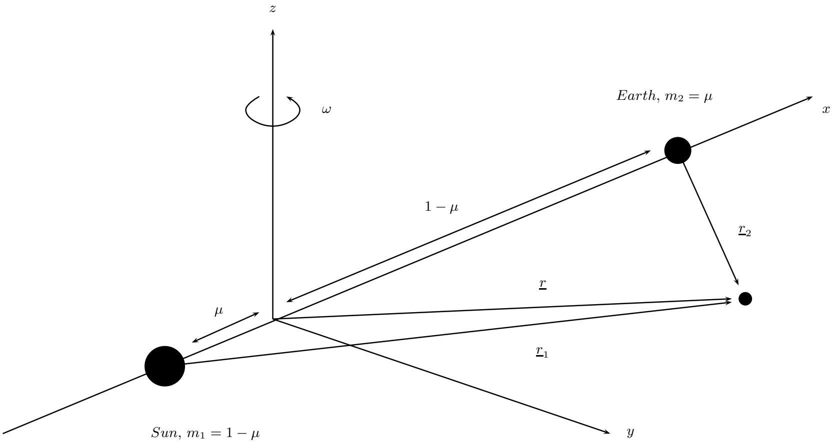
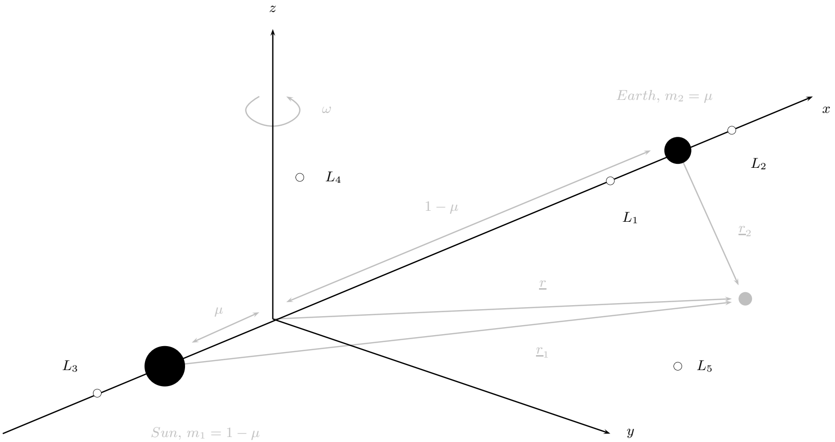
Lagrange points

We consider a **rotating coordinate frame** in which the primaries are fixed:



Lagrange points

There are five equilibrium points called **Lagrange points**. All are in the plane of the primaries' mutual orbit.



Sail model

- In the rotating coordinate frame the equations of motion of the solar sail are

$$\frac{d^2\underline{r}}{dt^2} + 2\underline{\theta} \times \frac{d\underline{r}}{dt} = \underline{a} - \underline{\theta} \times (\underline{\theta} \times \underline{r}) - \nabla V \equiv \underline{F}, \tag{1}$$

where

$$V = -\left(\frac{1-\mu}{|r_1|} + \frac{\mu}{|r_2|}\right) \quad \text{and} \quad \underline{a} = \beta \frac{1-\mu}{r_1^2} (\widehat{\underline{r}}_1 . \underline{n})^2 \underline{n}.$$

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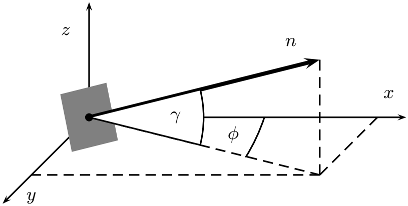
Sail model

- The free parameters are the lightness number β and the orientation of the unit normal $\underline{n}$.
- To place this anlaysis in the near-term we will use moderate values of $\beta \sim 0.05$, which corresponds to sail loading $\sigma \sim 30\ g/m^2$ or characteristic acceleration $a_0 \sim 0.3\ mm/s^2$.
- The orientation is given by the angles γ, ϕ which the normal makes with the rotating coordinate axes.



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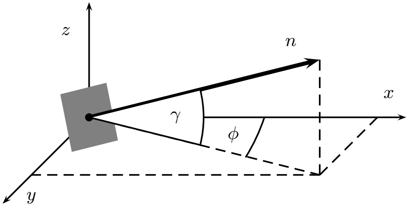
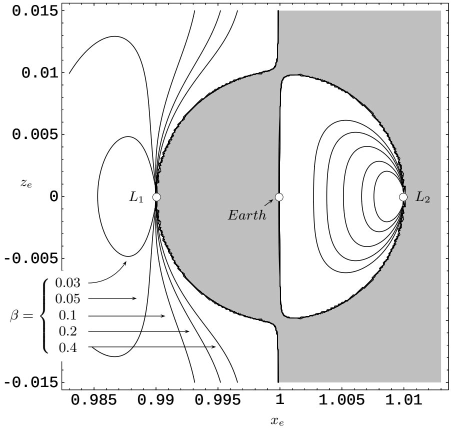


Fixed points

Fixed points, or equilibrium points, are given by the zeroes of $\underline{F}$ in (1). We find continuous surfaces in the x-z plane (sail angle $\phi = 0$), in fact a **two parameter family** given by the pair (β, γ):



Linear system

Letting $\underline{r} \to \underline{r}_e + \delta\underline{r}$ we Taylor expand $\underline{F}$. With $\delta\underline{r} = (x, y, z)^T$ the variation in the principal directions, we find to linear order

$$\begin{aligned} \ddot{x} - 2\dot{y} &= ax + bz \\ \ddot{y} + 2\dot{x} &= \quad cy \\ \ddot{z} \quad &= dx + ez \end{aligned}$$

We write this as a first order 6-d system by letting $X = (\delta r, \delta\dot{r})$ to give

$$\dot{X} = AX,$$

and the eigenvalues of A are

$$\{\pm\lambda_1 i, \pm\lambda_2 i, \pm\lambda_r\}$$

The linear eigenvalues are very informative as to the dynamics of the sail near the fixed point and beyond.

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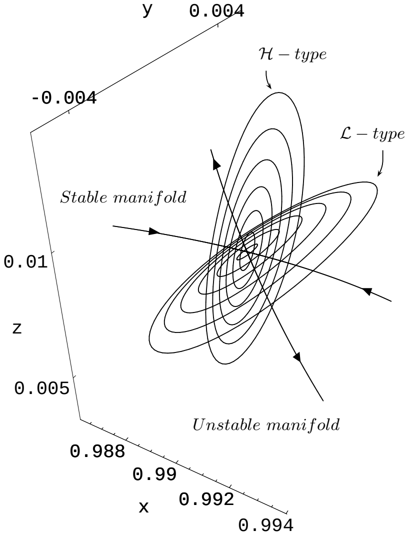
$$\{\pm\lambda_1 i, \pm\lambda_2 i, \pm\lambda_r\}$$

- There are two families of periodic orbits, one vertical ($\mathcal{H}$-type) and one horizontal ($\mathcal{L}$-type).
- There are trajectories which go onto and away from the fixed point, these are the stable and unstable manifolds resp.



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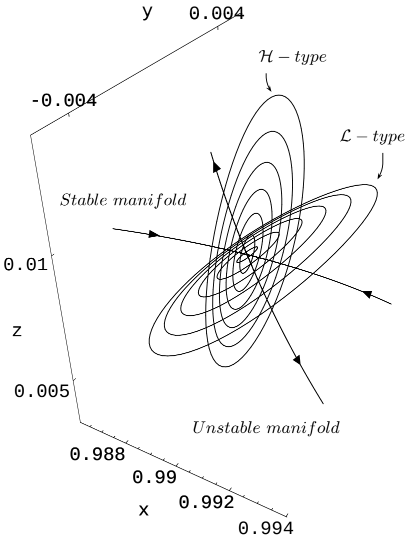

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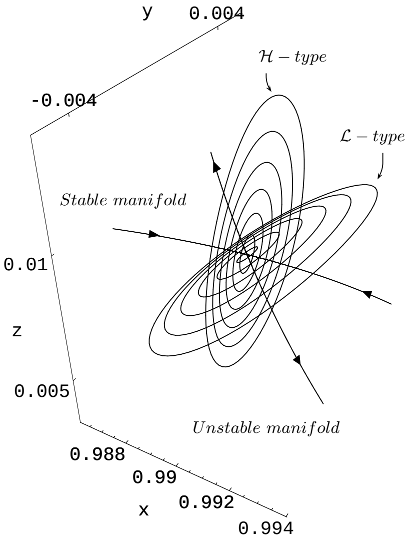

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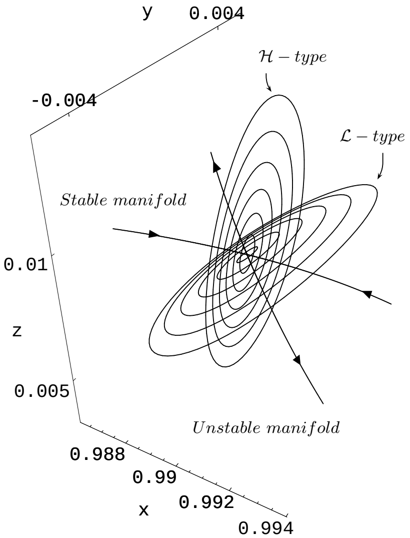
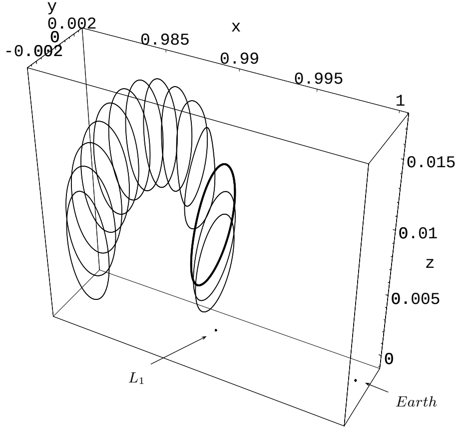


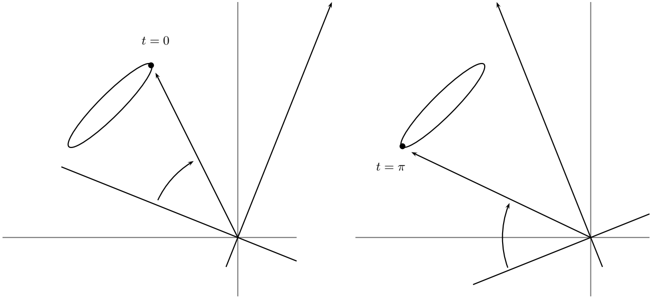
Large amplitude periodic orbits

There is a four parameter family of periodic orbits about fixed points in the x-z plane. For example, we show here a family of periodic orbits around a series of equilibria along $\beta = 0.05$.



Polesitter

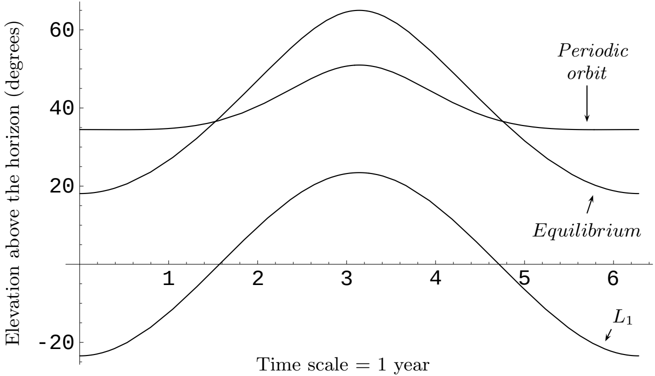
- Some orbits have a period close to one year. We may use these orbits to provide a constant view of one of the poles.



Wintertime, Northern Hemisphere *Summertime, Northern Hemisphere*

Polesitter

By timing the orbit about the fixed point well, we can narrow the angle of elevation of the sail when viewed from the pole, compared to a sail at the equilibrium point:



Circular restricted 3-body problem

Linearised system

Applications 1: Periodic orbits

Applications 2: Self-connected fixed points

Applications 3: Doubly-connected fixed points

Other interesting dynamics

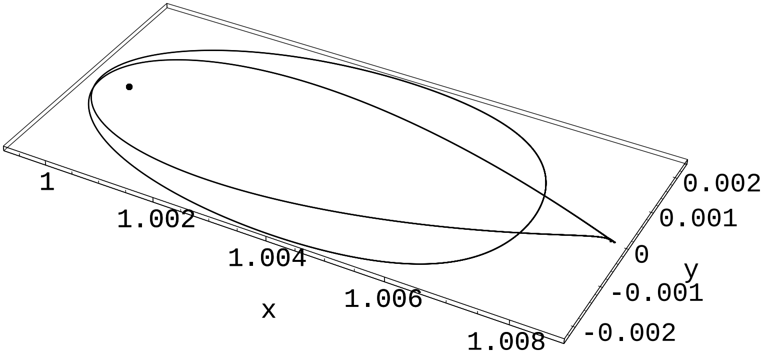
Self-connection

$$\{\pm\lambda_1 i, \pm\lambda_2 i, \pm\lambda_r\}$$

- As there is a positive *and* negative real eigenvalue, this means there is one trajectory going onto the fixed point and one going away.
- In some cases, these trajectories intersect smoothly, that is, they are the same.
- This means the dynamics of the system will naturally and freely bring the sail away from the fixed point only to bring it back onto the fixed point again (homoclinic path).

Self-connection

There are many examples of these self-connected fixed points, with different number loops of the Earth. Here's a double loop.



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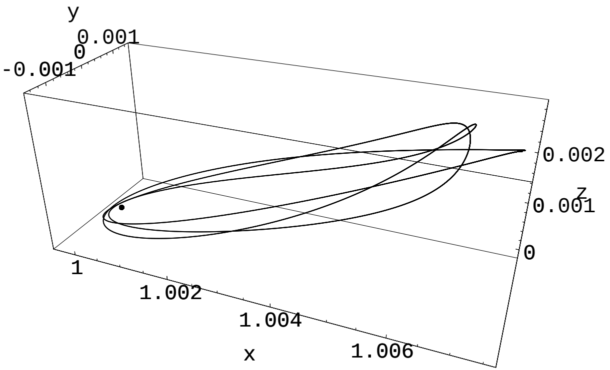
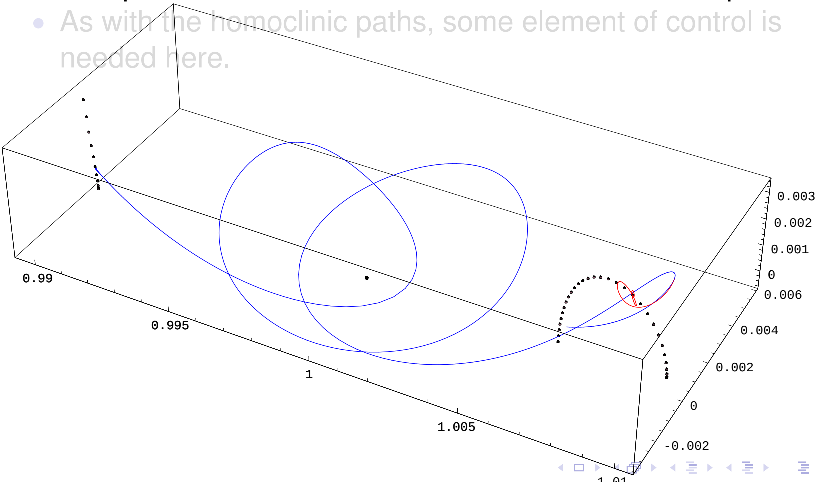


Double-connection

- There are some cases where the trajectory leaving one fixed point is identical to the trajectory entering another fixed point (heteroclinic path).
- This provides us with a ‘free’ transfer between fixed points.
- As with the homoclinic paths, some element of control is needed here.



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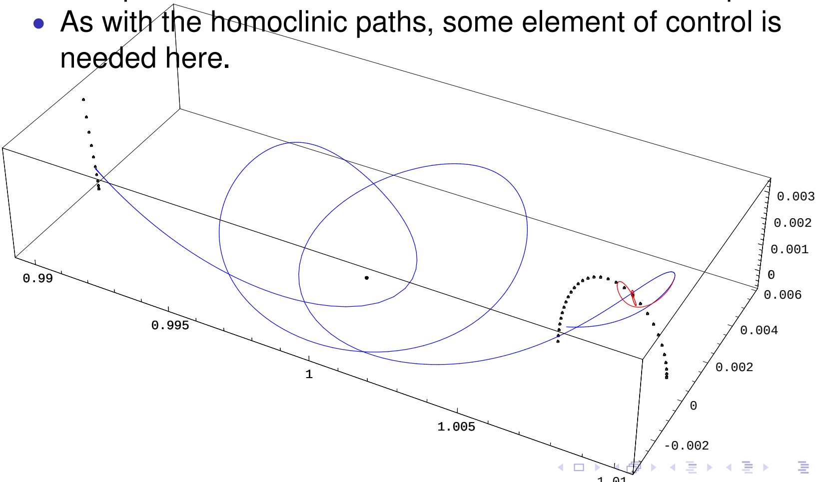

Homoclinic paths of periodic orbits

- Just like fixed points, periodic orbits about fixed points have a stable and unstable mode. These describe trajectories that wind onto and off of the periodic orbit.
- These are useful for transfer and self-connection.
- Interestingly, if a fixed point is self-connected, then so is the periodic orbit about it.
- In fact, *every* trajectory that winds off the periodic orbit will wind back onto it in the future. This feature is without analogue in the classical Earth-Sun case.

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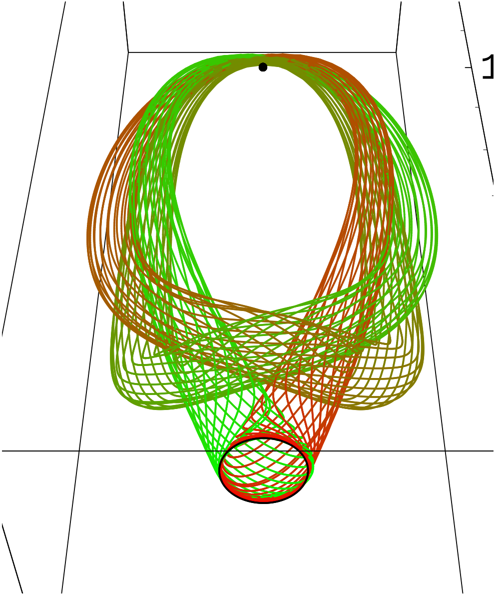
Homoclinic periodic orbits

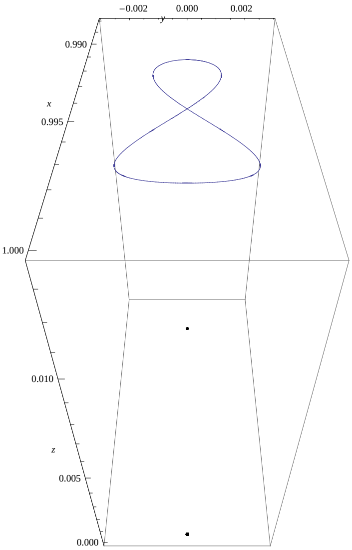


In other words, periodic orbits can inherit the homoclinic nature of the equilibrium point which they orbit.

Multiply-periodic orbits

$$\{\pm\lambda_1 i, \pm\lambda_2 i, \pm\lambda_r\}$$

- There are two continuously varying frequencies; there are therefore families of fixed points where the frequencies are in ratio, $\lambda_1/\lambda_2 \in \mathbb{Z}$.
- We may use these points to describe multiply-periodic orbits.



Summary

- Solar sails admit families of orbits beyond the Keplerian orbits of conventional spacecraft.
- There are continuous surfaces of fixed points in the solar sail 3-body problem.
- Fixed points in the x-z plane have the linear dynamical structure of centres crossed with saddles.
- We may therefore describe families of periodic orbits displaced above the ecliptic plane.
- We may also find self-connected and doubly-connected fixed points.
- Other interesting orbits arise with further investigation.