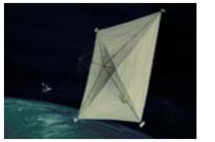

Analytical Model for Solar Sails with Applications

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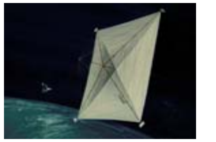
Outline

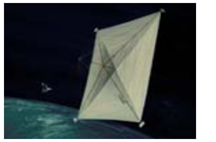
- Introduction
- Flat Solar Sail Model
- Derivation of the Generalized Sail Model (GSM)
  - Force Equation
  - Moment Equation
- Computation of GSM tensors
- Estimation of GSM Force and Moment
- Trajectory Control
  - Ideal Sail about halo orbit
  - GSM for station-keeping sail at sub-L1 point
- Conclusions
- Future work

Motivation

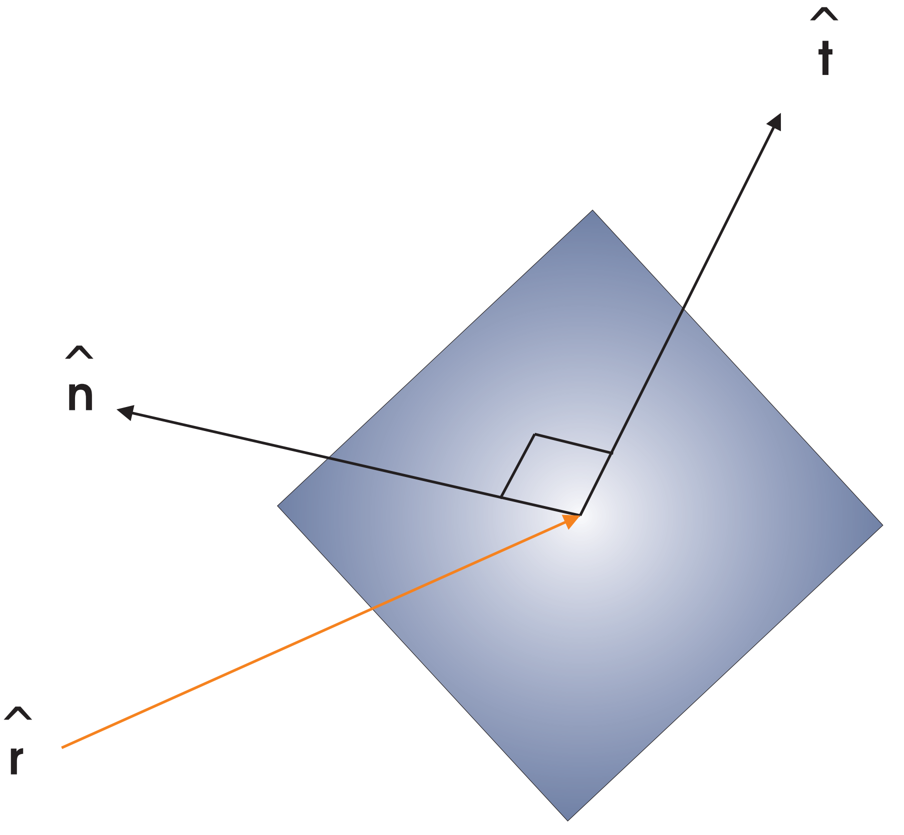


- To fly a solar sail a precise navigation system must be in place to model the forces and torques generated by the solar sail.
- The only traditional analytical sail model was for a flat sail. Unable to model sails with billow and wrinkles.
- Previously, the only way to model a complex sail was through the use of finite element models, which are difficult to introduce into the navigation process.
- The GSM provides an alternative to FEM to analytically model sails .
- Assumes that the sail shape is rigid and does not change with attitude.
- Assumes that there in no self-shadowing on the surface.

Non-Ideal Flat Sail Model



- $\hat{\mathbf{n}}$ is the sail normal vector.
- $\hat{\mathbf{r}}$ incidence radiation vector.
- $\hat{\mathbf{t}}$ transverse vector.



Generalized Sail Model

- Treat a sail differential area element as a flat, non-ideal surface with area dA.

$$
\begin{aligned}
d\mathbf{F}_n &= -P(r)\Big[(1+\rho s)\cos^2\alpha + B_f(1-s)\rho\cos\alpha + (1-\rho)\frac{\epsilon_f B_f - \epsilon_b B_b}{\epsilon_f + \epsilon_b}\cos\alpha\Big]\hat{\mathbf{n}}dA \\
d\mathbf{F}_t &= P(r)A(1-\rho s)\cos\alpha\sin\alpha\hat{\mathbf{t}}dA
\end{aligned}
$$

- The total force and moment are found by integrating

$$
\mathbf{F} = \int_A d\mathbf{F}, \quad \mathbf{M} = \int_A \vec{\varrho}\times d\mathbf{F}
$$

- Equivalence on cross product

$$
\varrho \times d\mathbf{F} = \tilde{\varrho}\cdot d\mathbf{F} \qquad \tilde{\varrho} = \begin{bmatrix} 0 & -\varrho_3 & \varrho_2 \\ \varrho_3 & 0 & -\varrho_1 \\ -\varrho_2 & -\varrho_1 & 0 \end{bmatrix}
$$

GSM Force

The force equation is given by

$$\mathbf{F} = PA\Big[\mathbf{J}^2 \cdot \hat{\mathbf{r}} - 2\hat{\mathbf{r}} \cdot \mathbf{J}^3 \cdot \hat{\mathbf{r}} - (\mathbf{J}^1 \cdot \hat{\mathbf{r}})\hat{\mathbf{r}}\Big]$$

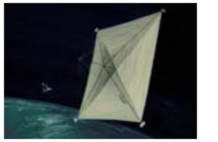
- Where the J^m tensors of rank-m are given by

$$\begin{aligned}
\mathbf{J}^1 &= \frac{1}{A}\int_A a_3 \hat{\mathbf{n}} dA \\
\mathbf{J}^2 &= \frac{1}{A}\int_A a_2 \hat{\mathbf{n}}\hat{\mathbf{n}} dA \\
\mathbf{J}^3 &= \frac{1}{A}\int_A \rho s \hat{\mathbf{n}}\hat{\mathbf{n}}\hat{\mathbf{n}} dA
\end{aligned}$$

- And the outer product of the sail normal is

$$\begin{aligned}
\hat{\mathbf{n}}\hat{\mathbf{n}} &= \hat{n}_i \hat{n}_j \\
\hat{\mathbf{n}}\hat{\mathbf{n}}\hat{\mathbf{n}} &= \hat{n}_i \hat{n}_j \hat{n}_k
\end{aligned}$$

GSM Moment



- Similarly, the moment equation is

$$\mathbf{M} \quad = \quad PAl_r\Big[\mathbf{K}^2 \cdot \hat{\mathbf{r}} - \hat{\mathbf{r}} \cdot \mathbf{K}^3 \cdot \hat{\mathbf{r}}\Big]$$

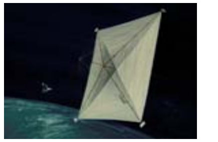
- Where the moment tensors are defined as

$$\begin{aligned}
\mathbf{K}^2 \quad &= \quad \frac{1}{Al_r}\int_A a_2 \tilde{\varrho} \cdot \hat{\mathbf{n}}\hat{\mathbf{n}} dA \\
\mathbf{K}^3 \quad &= \quad \frac{1}{Al_r}\int_A \Big[\rho s\Big(-2\tilde{\varrho} \cdot \hat{\mathbf{n}}\hat{\mathbf{n}}\hat{\mathbf{n}} + \tilde{\varrho} \cdot \hat{\mathbf{n}}\overline{\overline{\mathbf{U}}}\Big) - \tilde{\varrho} \cdot \hat{\mathbf{n}}\overline{\overline{\mathbf{U}}}\Big]\, dA
\end{aligned}$$

Properties of GSM Tensors

- All J and K tensors are function of the sail geometry and sail optical parameters.
- Can be computed off-line and used over a wide range of sail attitudes.
- Force Tensors
  - Force tensors are symmetric in all their indices, i.e. $J_{ijk} = J_{ikj}$, etc.
  - J^1, J^2, and J^3 require 3, 6, and 10 independent coefficients.
  - The force acting on a solar sail of arbitrary shape is characterized by at most 19 coefficients.
- Moment Tensors
  - Moment tensors are not symmetric in general.
  - Only K^3 presents some symmetries in its two last indices. $K^3_{\ ijk} = K^3_{\ ikj}$
  - 27 coefficients are needed to capture the moment acting on a solar sail.

Spherical Sail



- One Hemisphere illuminated
- Normal Vector depends on location
- Surface equation

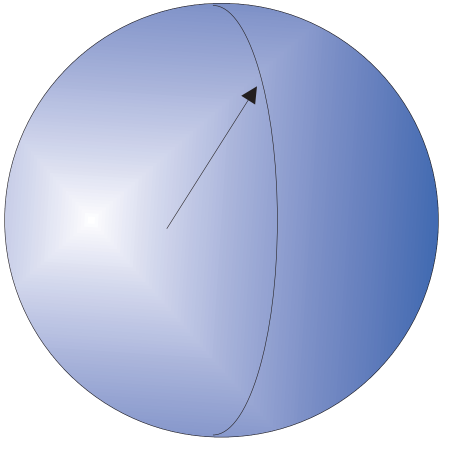
$$\Phi = x^2 + y^2 + z^2 - r^2 = 0$$

- Normal vector given by:

$$\hat{\mathbf{n}} = \frac{\nabla\Phi}{\|\nabla\Phi\|} = \frac{1}{r}\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

- Using polar coordinates to perform integration.

$$dA = r^2 \cos\phi d\phi d\theta$$



Spherical Sail

- Force coefficients

$$\mathbf{J}^1 = a_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\mathbf{J}^2 = a_2 \frac{2}{3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{J}^3_{ij1} = \rho s \frac{1}{4} \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

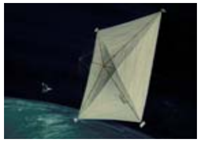
$$\mathbf{J}^3_{ij2} = \rho s \frac{1}{4} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\mathbf{J}^3_{ij3} = \rho s \frac{1}{4} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

- Force equation

$$\mathbf{F} = P(r)A\Big[- a_2 \frac{2}{3} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - \rho s \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - a_3 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Big]$$

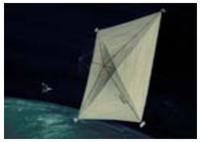
Estimation of GSM Tensor Coefficients



- Challenges for Sail Navigation:
  - It is difficult to precisely determine the sail propulsion performance.
  - Despite this, precise navigation performance is needed for the success of a solar sail mission.
  - Refinement of the force and moment model must be done with in-flight data.
  - The GSM force and moment equations are linear in the force and moment tensor coefficients. Thus, it is possible to develop a linear estimation method to refine models of the sail force and moment.
  - In the following we use normalized force and moment coefficients. Derived from measured accelerations and angular rates.

$$\mathbf{C}_F = \frac{\mathbf{F}}{PA} \qquad \mathbf{C}_M = \frac{\mathbf{M}}{PAl}$$

Linear Force and Moment



- The GSM normalized force and moment are:

$$\begin{aligned}\mathbf{F}_c &= \mathbf{J}^2 \cdot \hat{\mathbf{r}} - 2\hat{\mathbf{r}} \cdot \mathbf{J}^3 \cdot \hat{\mathbf{r}} - (\mathbf{J}^1 \cdot \hat{\mathbf{r}})\hat{\mathbf{r}} \\ \mathbf{M}_c &= \mathbf{K}^2 \cdot \hat{\mathbf{r}} - \hat{\mathbf{r}} \cdot \mathbf{K}^3 \cdot \hat{\mathbf{r}}\end{aligned}$$

  - GSM equations are not in a form to facilitate the estimation and must be manipulated into a linear form.
- The force equation can be written as

$$\mathbf{F}_c = \mathbf{A}_F(\hat{\mathbf{r}})\mathbf{J}$$

  - Where $\mathbf{A}_F$ is a 3x19 matrix that contains the known sail attitude relative to the sun.
  - **J** is the vector that contains the unknown force parameters.
- Similarly for the moment equation

$$\mathbf{M}_c = \mathbf{A}_M(\hat{\mathbf{r}})\mathbf{K}$$

  - Where $\mathbf{A}_M$ is a 3X27 matrix.
  - **K** is the vector that contains the moment coefficients.

Estimation of GSM Tensor Coefficients

- Cost-Function:

$$V = \frac{1}{2} \sum_{j=1}^{N} \left(\mathbf{y}_j - \mathbf{A}_j \mathbf{x}\right)^T P_{cc}^{-1} \left(\mathbf{y}_j - \mathbf{A}_j \mathbf{x}\right)$$

  - y_j are the normalized force or moment measurements (or given coefficients) at an attitude corresponding to $\hat{\mathbf{r}}_j$.
  - P_{cc} is the measurement covariance.

- Solving the least-squares problem

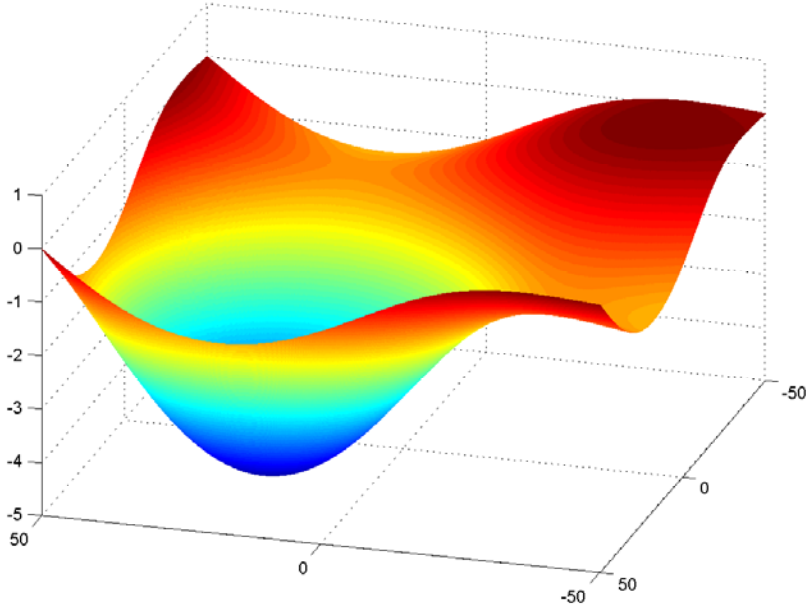
$$\mathbf{x} = P_{xx} \sum_{j=1}^{N} \mathbf{A}_j^T P_{cc}^{-1} \mathbf{y}_j$$

- where

$$P_{xx}^{-1} = \sum_{j=1}^{N} \mathbf{A}_j^T P_{cc}^{-1} \mathbf{A}_j$$

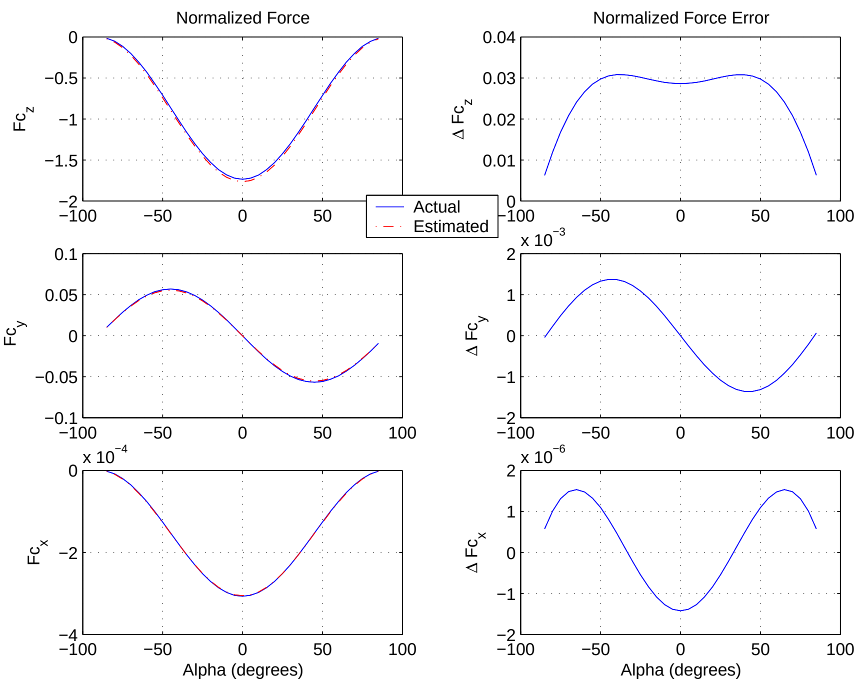
Attitude Sampling Estimation

- Sinusoid sail used as baseline due to non-symmetric shape.
- GSM tensors computed numerically for this shape.

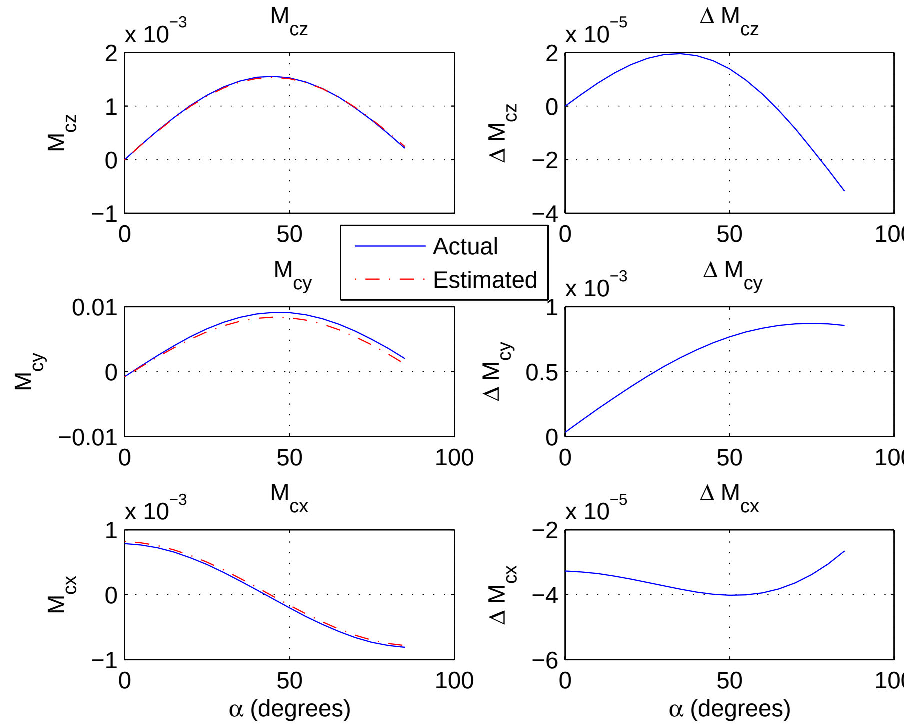


Force and Moment Estimation

Force



Moment



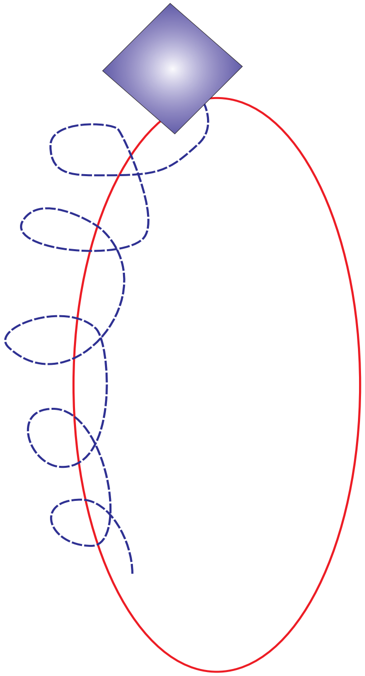
Control Problem Definition

- Due to the difficulty of predicting the sail propulsion on the ground, it is likely that solar sail missions are determined using a conservative model of sail propulsion to allow for uncertainties in the sail performance.
  - If the flown sail has lower performance than the mission requires it would lead to a failure
  - If the flown sail has exactly the performance required, the mission can be achieved with small margin for errors.
  - If the flown sail has excess performance, the mission can be achieved. Maneuvering is required.

- Despite this, it is desired to achieve the original sail mission being a trajectory or station-keeping.

Control Problem Definition

- Here, we focus on maintaining a sail with excess performance orbiting a sub-L_1 halo orbit.
- Use excess force to 'orbit' nominal trajectory.



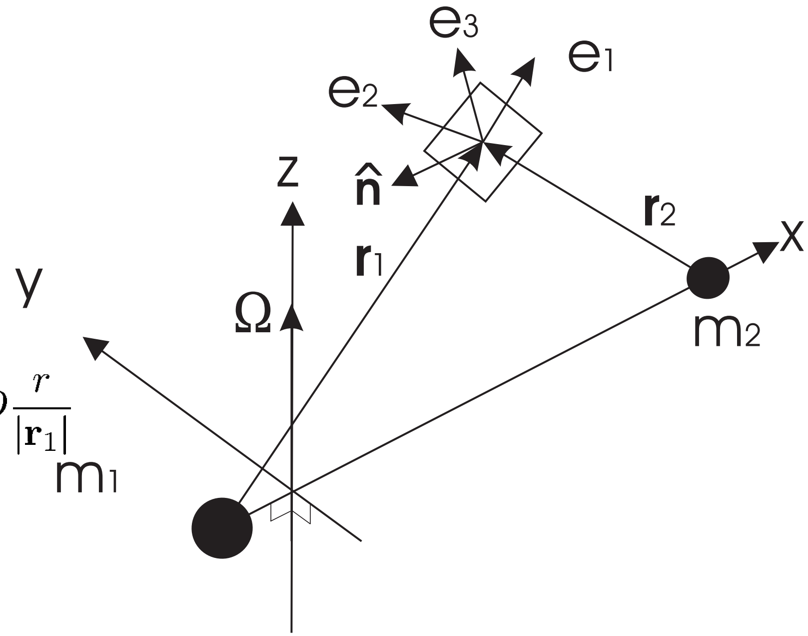
Circular Restricted 3-Body Problem

- Equation of motion written in scalar components

$$
\begin{aligned}
\ddot{x} &= 2\dot{r}\cos\theta - 2r\dot{\theta}\sin\theta + x - \frac{(1-\mu)(x+\mu)}{|\mathbf{r}_1|^3} - \frac{\mu}{|\mathbf{r}_2|^3}(x+\mu-1) + a_x \\
\ddot{r} &= -2\dot{x}\cos\theta + r\cos^2\theta - \left(\frac{1-\mu}{|\mathbf{r}_1|^3} + \frac{\mu}{|\mathbf{r}_2|^3}\right)r + r\dot{\theta}^2 + a_r \\
\ddot{\theta} &= 2\dot{x}\frac{\sin\theta}{r} - \sin\theta\cos\theta - 2\frac{\dot{r}\dot{\theta}}{r} + \frac{a_\theta}{r}
\end{aligned}
$$

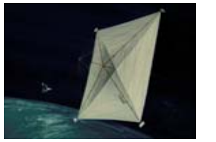
- Acceleration of an ideal sail

$$
\begin{bmatrix} a_x \\ a_r \\ a_\theta \end{bmatrix} = \frac{\beta(1-\mu)}{|\mathbf{r}_1|^2}\cos^2\alpha \begin{bmatrix} \cos\alpha \\ -\sin\alpha\cos(\delta-\theta) \\ -\sin\alpha\sin(\delta-\theta) \end{bmatrix} + O\frac{r}{|\mathbf{r}_1|}
$$



- b sail lightness, $\hat{\mathbf{n}}$ sail normal vector

Control of Sail x-location



- Active Proportional derivative controller

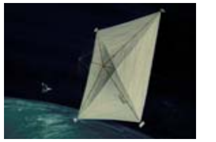
$$\alpha = \alpha_1 + c_1(x - x_{L_1}) + c_2\dot{x}$$

- where

$$\cos^3\alpha_1 \quad = \quad -\left[2\dot{r}\cos\theta - 2r\dot{\theta}\sin\theta + x_{L_1} - \frac{(1-\mu)(x_{L_1}+\mu)}{|\mathbf{r}_1(x_{L_1})|^3} - \frac{\mu}{|\mathbf{r}_2(x_{L_1})|^3}(x_{L_1}+\mu-1)\right]\frac{|\mathbf{r}_1(x_{L_1})|^2}{\beta'(1-\mu)}$$

- c_1 and c_2 are the gains of the PD-controller

Control of about Halo Orbit



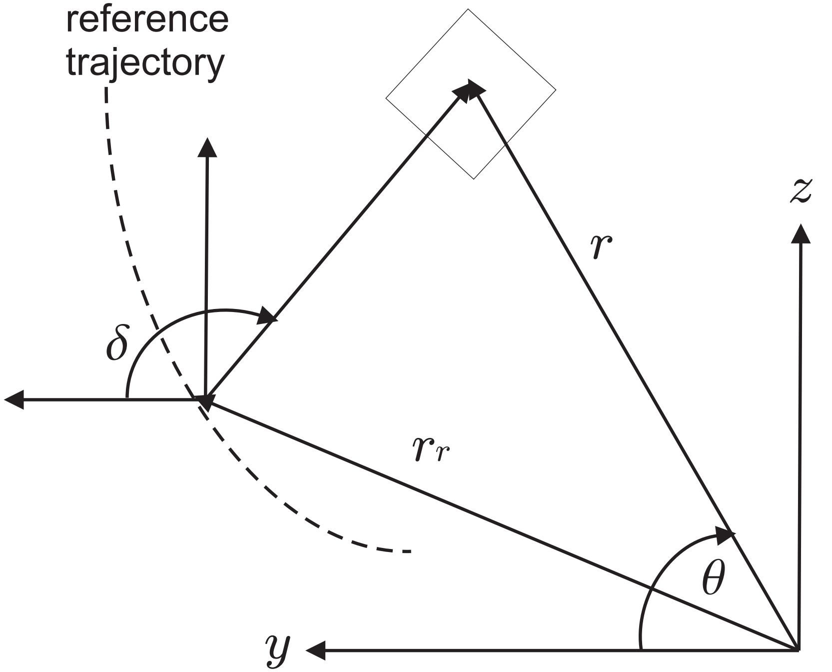
- Halo orbit treated as a series of moving point

- Controller for orbiting orbit

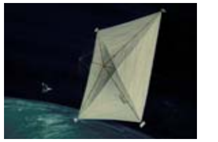
$$\overline{\delta} = \delta - \theta - \delta_u$$

$$\begin{aligned}
\tan\delta &= \frac{z - z_r}{y - y_r} \\
\tan\theta &= \frac{z}{y} \\
\sin\delta_u &= \frac{E_r - E_0}{k_2}
\end{aligned}$$

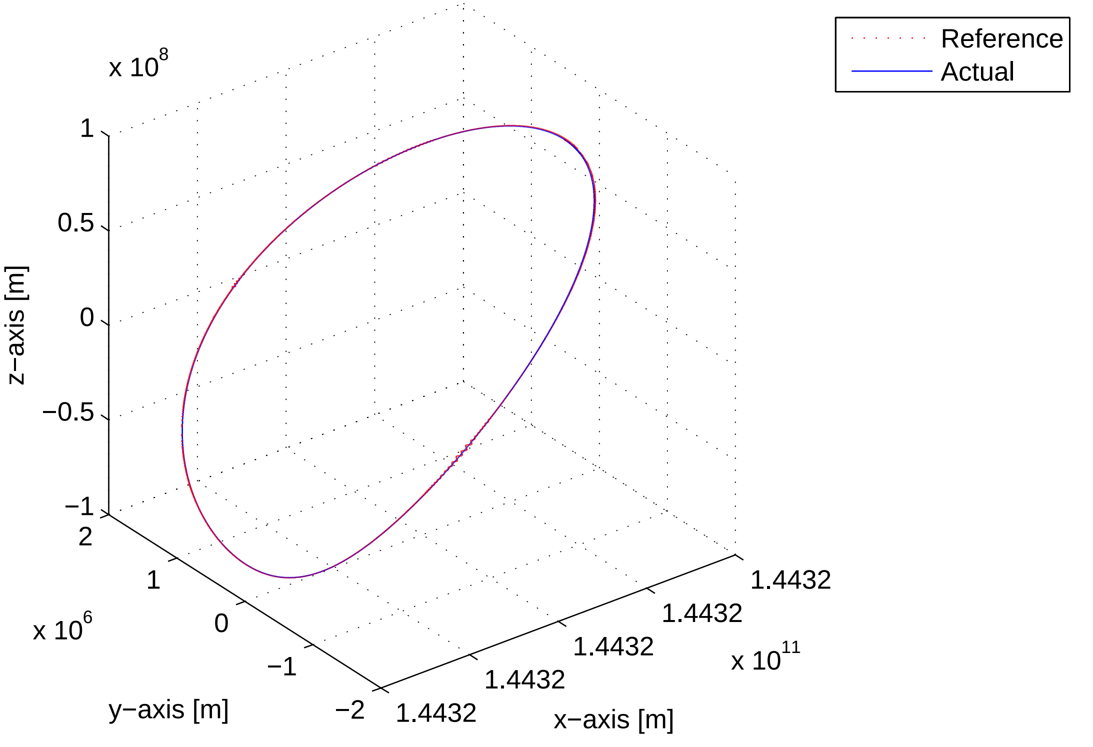
$$E = \frac{1}{2}\left(r^2\dot{\theta}^2 + \dot{r}^2\right)$$



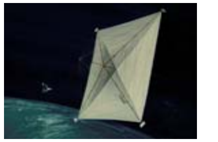
Control of about Halo Orbit



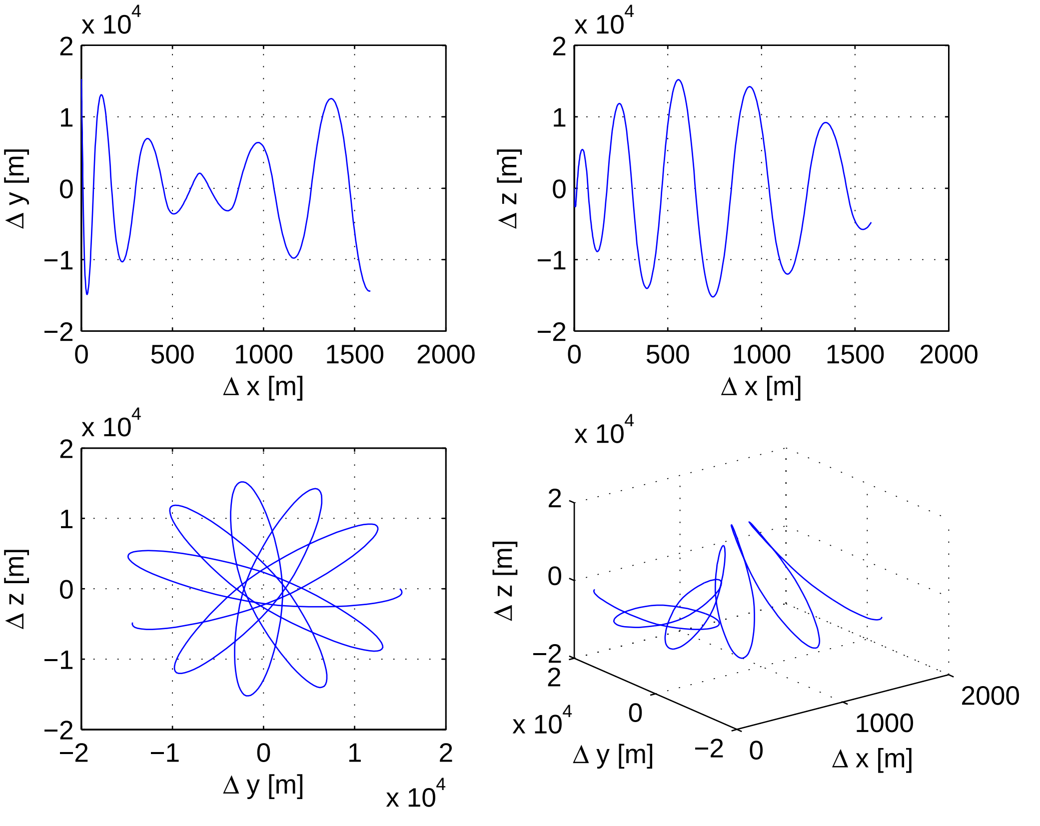
- Halo orbit obtained from approximate series solution.



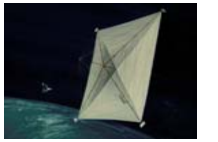
Control of about Halo Orbit



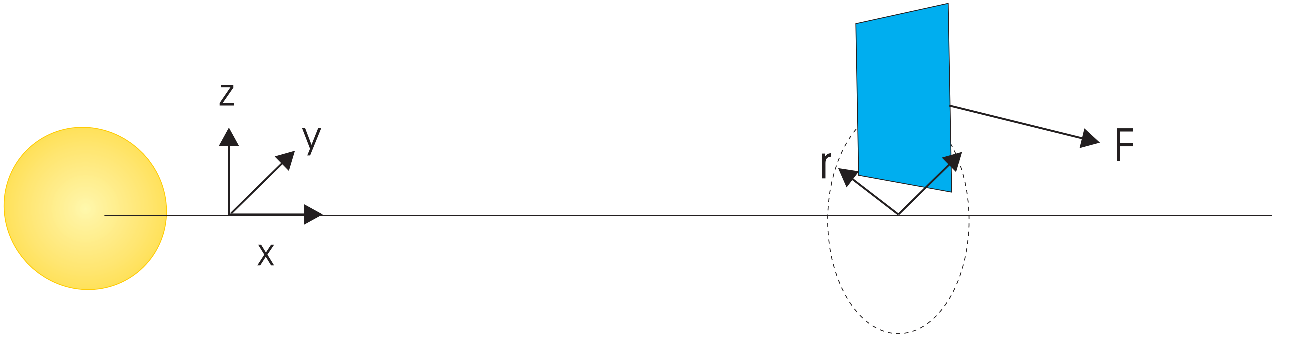
- Sail relative position with respect to moving point on halo orbit



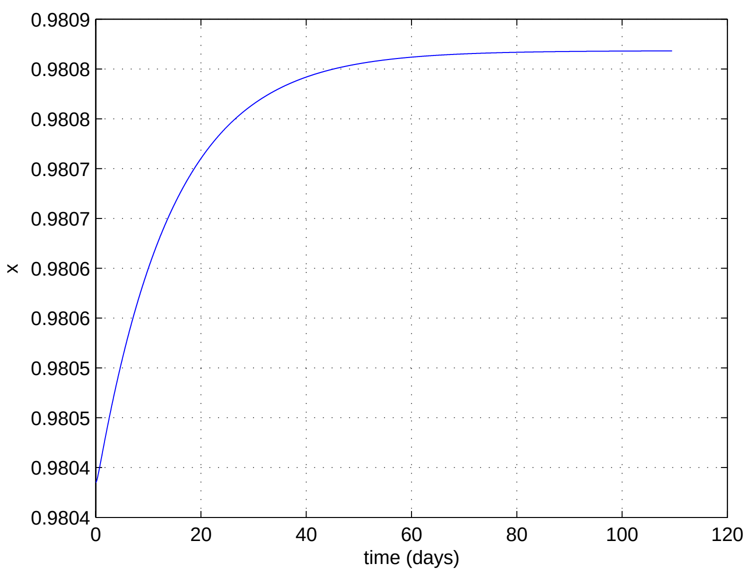
Trajectory Controller for Billow Sail

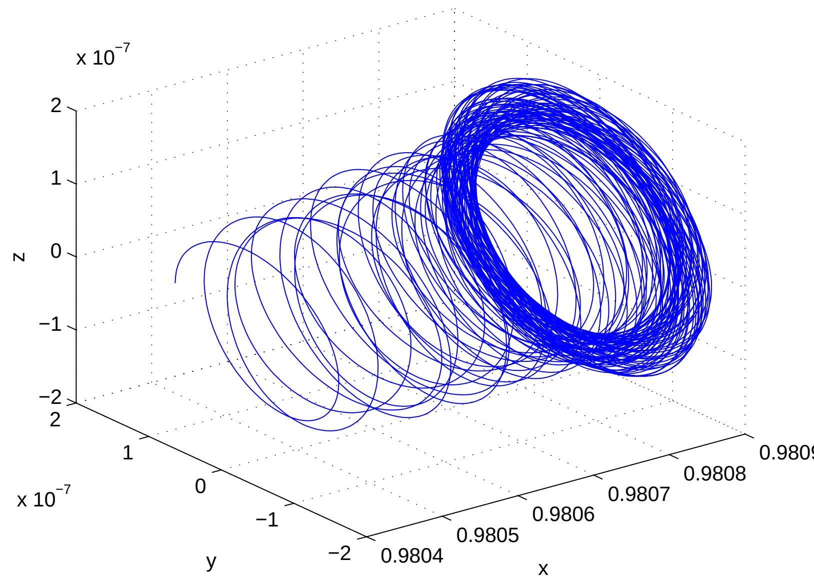


- Trajectory controller simplifies for sub-L1 point.
- Used the excess force to cancel the centripetal acceleration.



Control of about Halo Orbit





Conclusions

- The GSM is an analytic model for the force and moment generated by a solar sail as function of sail attitude.
- The GSM can characterize solar sails of arbitrary shape with non-ideal optical parameters that are function of the sail surface.
- Since the GSM is an analytic result, it can be applied wherever the flat sail model has been used.
- A general approach for estimating the GSM force and moment tensor coefficients was defined and tested.
- A trajectory controller for ideal sails was shown for practical missions.

Future work

- Future research will investigate:
  - Modification of the GSM for non-constant shapes that change with attitude.
  - Include self shadowing effects on the GSM.
  - Optimization of force and moment estimation
  - Study excess-thrust controller for general sails orbiting halo orbits
  - Implement GSM into trajectory control